Northwestern University

Name: ____________________

Math 230, Midterm
Summer 2012
July 16, 2012

Prob.	Points	Score
1	10	
2	20	
3	20	
4	10	
5	10	
6	10	
7	10	
8	10	
Total	100	

Instrctions:

- You have 90 minutes to complete the exam.
- You may not use books, notes or calculators.
- Read each question carefully.
- Write legibly and show your work. Feel free to use both sides of each page.
- Ask for additional paper if you need.
- This exam has 11 pages and 8 problems. Please make sure that all pages are included.

GOOD LUCK !

Question 1. (10 points) True or False? Circle the correct answer. No justification needed.

(a) If $\vec{a}$ and $\vec{b}$ are non-zero parallel vectors then for any vector $\vec{c}$, $|\text{comp}_{\vec{a}}\vec{c}| = |\text{comp}_{\vec{b}}\vec{c}|$.

True **False**

(b) For any two vectors $\vec{a}$ and $\vec{b}$, $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$ and $\vec{a}\times\vec{b} = \vec{b}\times\vec{a}$.

True **False**

(c) The point $(-4, 0)$ in rectangular coordinates has $(-4, 3\pi)$ as one of its polar coordinate representations.

True **False**

(d) If $\vec{r}(s)$ is the arc length parametrization of a curve with a given starting point and a direction, then $\vec{r}\,'(s) = \dfrac{d}{ds}\vec{r}(s)$ is the unit tangent and $||\vec{r}\,''(s)|| = \left|\left|\dfrac{d^2}{ds^2}\vec{r}(s)\right|\right|$ is the curvature to the curve at $\vec{r}(s)$.

True **False**

(e) The equation $x^2 + y^2 = 1$ represents a circle in both $\mathbb{R}^2$ and $\mathbb{R}^3$.

True **False**

Problem 2. (20 Points) Let $P = (1, 0, 0)$, $Q = (2, 1, 1)$, and $R = (0, 1, 1)$. Answer the following.

(a) Find the cosine of the angle between $\overrightarrow{a} = \overrightarrow{PQ}$ and $\overrightarrow{b} = \overrightarrow{PR}$.

(b) Find vectors $\overrightarrow{u}$, $\overrightarrow{v}$ such that $\overrightarrow{u} + \overrightarrow{v} = \overrightarrow{a}$, $\overrightarrow{u} \parallel \overrightarrow{b}$ and $\overrightarrow{v} \perp \overrightarrow{b}$. (Hint. use projection)

Continuation of problem 2 from previous page, points $P = (1, 0, 0)$, $Q = (2, 1, 1)$ and $R = (0, 1, 1)$.

(c) Find the area of the the triangle $\triangle PQR$.

(d) Find the equation of the sphere with center P and passing through R.

Question 3. (20 points) Line L_1 is given by the symmetric equations $\dfrac{x-1}{2} = -y = \dfrac{z+2}{3}$.

(a) Find the parametric equation for the line L_2 which is parallel to L_1, and passes through $(2, 1, -1)$.

(b) Find the equation of the plane $\mathcal{P}$ containing the lines L_1 and L_2.

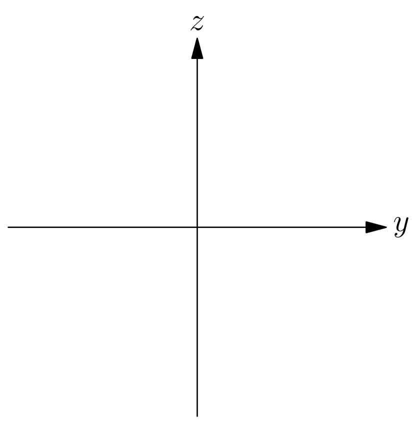
Continuation of problem 3 from previous page.

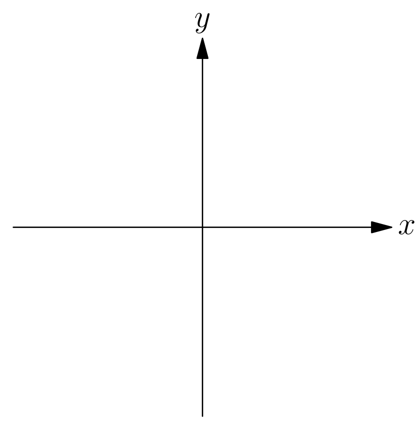
(c) What is the distance between the point $(1, 1, 1)$ and plane $\mathcal{P}$?

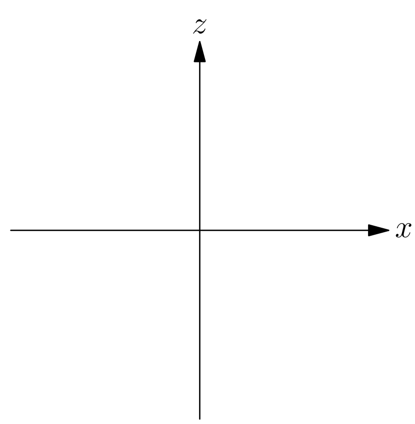
(d) Find the point on plane $\mathcal{P}$ closest to $(1, 1, 1)$. (It is the point where the line perpendicular to $\mathcal{P}$ passing throught $(1, 1, 1)$ intersects the plane $\mathcal{P}$.)

Question 4. (10 points) Consider the surface given by the equation $-x^2 + 4y^2 - z^2 = 4$.

(a) Sketch the $x = 0$, $z = 0$ and $y = \sqrt{2}$ traces on the graphs below. Clearly label the intercepts (points where the curves meet the axes).







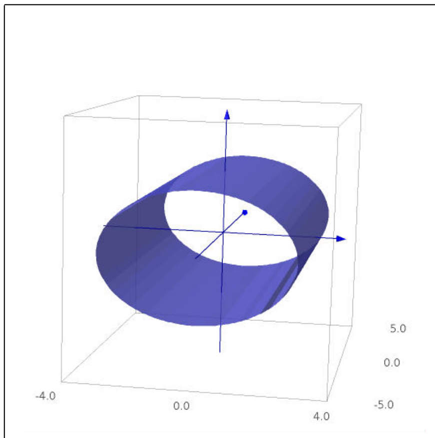
(b) What is the $y = 0$ trace?

(c) What is the surface called and which axis does it open along?

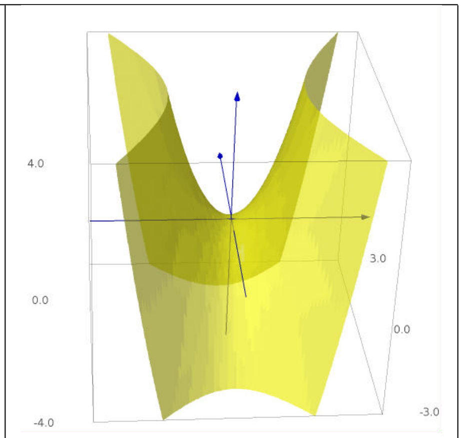
Question 5. (10 points) Match the following surfaces with their graphs:

(a) $z = \cos\left(\sqrt{x^2+y^2}\right)$ (b) $-x^2+4y^2-z^2=4$ (c) $4x^2+9y^2+9z^2=36$

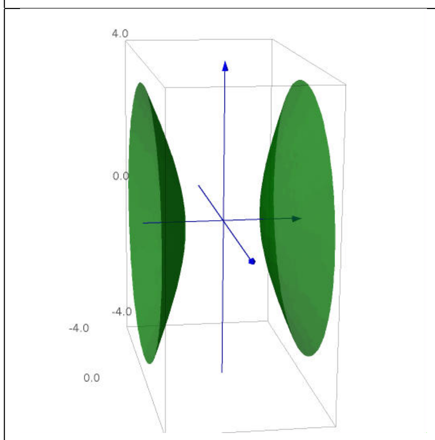
(d) $4x^2+9z^2=36$ (e) $z=2x^2-y^2$



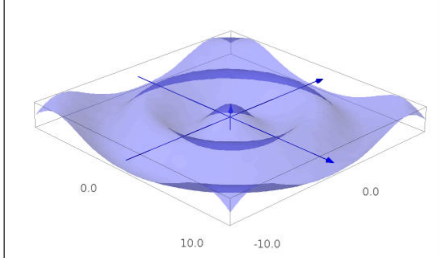
Surface ____________



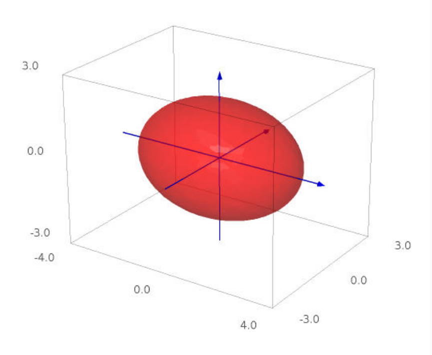
Surface ____________



Surface ____________



Surface ____________



Surface ____________

Question 6. (10 points) Consider the curve in $\mathbb{R}^2$ given by the equation $r = e^\theta$ in polar coordinates.

(a) Write down the parametric equation for the curve in rectangular coordinates with parameter θ.

(b) What is the tangent line to the curve at the point where $\theta = \pi$?

(c) Find the length of the curve between $t = 0$ and $t = \ln\left(5\sqrt{2} + 1\right)$.

Question 7 (10 points) Consider particle 1 moving in space with acceleration

$$\vec{a}(t) = \langle -6t,\ -4\pi^2\cos(2\pi t),\ -\pi^2\sin\left(\frac{\pi}{2}t\right)\rangle$$

It has initial position $\langle 0,1,0\rangle$ and initial velocity $\langle -1,1,2\pi\rangle$.

(a) Find the position function $\vec{r_1}(t)$ of the particle.

(b) Does it collide with particle 2 whose position function is $\vec{r_2}(t) = \langle\ -2t^2,\ t+t^2,\ 4t^3\ \rangle$?

Question 8 (10 points) The following parametric curve is a spherical spiral

$$\vec{r}(t) = \langle \sin(t)\cos(4t),\ \sin(t)\sin(4t),\ \cos t \rangle \qquad 0 \le t \le \pi$$

(a) Note that $\vec{r}\left(\frac{\pi}{2}\right) = \langle 1, 0, 0 \rangle$. What are the tangent, and unit tangent vectors to the curve at $(1, 0, 0)$?

(b) Calculate the curvature of the curve at $(1, 0, 0)$. (Choose the formula that will minimize calculation.)